

Notation.

- Let P be a d -dimensional simple polytope.
- Let α_i , $i = 0, 1, \dots, d - 1$ be the number of its i -dimensional faces (i -faces for short).
- For any face f of P denote by α_i^f the number of its i -faces.
- Let $\alpha_k^{(i)}$ be the average number of i -faces of a k -face of P :

$$\alpha_k^{(i)} = \frac{1}{\alpha_k} \sum_{\dim f=k} \alpha_i^f.$$

For any simple, compact, convex polytope $P \subset \mathbb{E}^d$ and any $i < k \leq [d/2]$ holds

$$\alpha_k^i < \binom{d-i}{d-k} \frac{\binom{[d/2]}{i} + \binom{[(d+1)/2]}{i}}{\binom{[d/2]}{k} + \binom{[(d+1)/2]}{k}}.$$

[Nik] V. V. Nikulin, *On the classification of arithmetic groups generated by reflections in Lobachevsky spaces*. Math. USSR Izv. 18 (1982), 99–123.